Random Union, Intersection and Membership

Douglas Cenzer

Department of Mathematics
University of Florida

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Random Reals

- Space $\mathcal{X} = \{0,1\}^{\mathbb{N}}$–reals
  Computable measure μ on X, such as Lebesgue (coin-toss) measure λ
  Biased (Bernoulli) coin toss λ_p – probability p of "1".
- For $\sigma \in \{0,1\}^*$, $I[\sigma] \subset \{0,1\}^{\mathbb{N}}$ denotes the set of infinite extensions of σ.
- For any $\sigma \in \{0,1\}^n$, $\lambda_a(I[\sigma]) = a^r(1-a)^{n-r}$, where $r = card(\{i : \sigma(i) = 1\})$.
- $X \in \mathcal{X}$ is μ-random if it passes all (c.e.) μ-Martin-Lof Tests$\{G_n : n \in \mathbb{N}\}$
- $\mu(G_n) \leq 2^{-n}$ implies $X \notin \bigcap_n G_n$.
- *A* is random *relative to B* if *A* passes all Martin-Löf tests which are c.e. in *B*.

Random Closed Sets

- $\mathcal{C}$ is the space of nonenmpty closed subsets of $\{0,1\}^{\mathbb{N}}$.
- (JLC 2007) A set in $\mathcal{C}$ is represented by a unique $X \in \{0,1,2\}^{\mathbb{N}}$ as follows.
- Let T_Q be the tree $\{\sigma : I[\sigma] \cap Q \neq \emptyset\}$.
  Let $\sigma_0 = \emptyset, \sigma_1, \dots$ enumerate T_Q.
  Let $X(n) = 2$ if $\sigma_n{}^\frown 0$ and $\sigma_n{}^\frown 1$ are both in T_Q and $X(n) = i < 2$ if $\sigma_n{}^\frown i \in T_Q$ but $\sigma_n{}^\frown (1-i) \notin T_Q$.
- This gives a computable homeomorphism between the space $\{0,1,2\}^{\mathbb{N}}$ and the space $\mathcal{C}$ of nonempty closed sets.
- For any measure μ on $\{0,1,2\}^{\mathbb{N}}$ and any $\mathcal{B} \subseteq \mathcal{C}$, let $\mu^*(\mathcal{B}) = \mu(\{X : Q_X \in \mathcal{B}\})$.
- λ_{a_1,a_2} gives probability a_i to $i < 2$. Lebesgue measure $\lambda = \lambda_{\frac{1}{3},\frac{1}{3}}$.
- The closed set Q_X is μ^*-random IFF X is μ-random.
- Two alternative characterizations will be discussed below.

Varying notions of randomness

- The main goal is to determine properties of random closed sets and their members.
- Another goal is to consider *different* notions of randomness *simultaneously*
- We will give two motivating examples:
- EXAMPLE I: Let $A, B \subseteq \mathbb{N}$. By van Lambalgen's Theorem, A and B are relatively random IFF $A \oplus B$ is random.
- $A \oplus B = \{2n : n \in A\} \cup \{2n+1 : n \in B\}$; for any C, let $C_0 = \{n : 2n \in C\}$ and $C_1 = \{n : 2n+1 \in C\}$, so $C = C_0 \oplus C_1$.
- $A \cup B$ is *not* random, since it has asymptotic density $\frac{3}{4}$. Likewise, $A \cap B$ is not random, since it has density $\frac{1}{4}$.
- THEOREM 1: If A and B are relatively λ-random, then
  - (a) $A \cup B$ is $\lambda_{\frac{3}{4}}$-random.
  - (b) $A \cap B$ is $\lambda_{\frac{1}{4}}$-random.

Proof of Theorem 1

- Let A and B be relatively random, so $A \oplus B$ is random.
- Given a Martin-Löf test $\{G_n : n \in \mathbb{N}\}$ for $A \cup B$, we convert it into a Martin-Löf -test $\{V_n\}$ for $A \oplus B$ as follows.
- Let $F(C) = C_0 \cup C_1$ and let $V_n = F^{-1}(G_n)$.
- This will be uniformly c.e. since F is computable and $\{G_n : n \in \mathbb{N}\}$ is uniformly c.e.
- LEMMA 1: For any Borel $G \subseteq \{0,1\}^{\mathbb{N}}$, $\lambda_{\frac{3}{4}}(G) = \lambda(F^{-1}(G))$.
- Now suppose that A and B are relatively random.
- Then for any Martin-Löf test $\{G_n : n \in \mathbb{N}\}$ with $\lambda_{\frac{3}{4}}(G_n) < 2^{-n}$, we have $\lambda(F^{-1}(G_n)) < 2^{-n}$.
- Thus, for some n, $A \oplus B \notin V_n = F^{-1}(G_n)$ and hence $A \cup B = F(A \oplus B) \notin G_n$.
- Hence $A \cup B$ is $\lambda_{\frac{3}{4}}$-random.
- The proof for $A \cap B$ is similar.

Example II

- Let Q be a random closed set (for Lebesgue measure).
- Being closed, Q must have a leftmost element L_Q and a rightmost element R_Q.
- L_Q and R_Q are *not* random (for Lebesgue measure)– it is known that L_Q has asymptotic density $\frac{1}{3}$ and that R_Q has asymptotic density $\frac{2}{3}$.
- THEOREM 2 [JLC07] If the closed set Q is Martin-Löf -random, then the leftmost element L_Q of Q is $\lambda_{\frac{1}{3}}$-random and the rightmost element R_Q is $\lambda_{\frac{2}{3}}$-random.
- A natural question for an arbitrary measure μ on $\{0,1\}^{\mathbb{N}}$ is whether any (some?) random closed set Q contains a μ-random element.

Proof of Theorem 2

- We sketch the proof for L_Q. Let $\nu = \lambda_{\frac{1}{3}}$.
- For $U \subseteq \{0,1\}^{\mathbb{N}}$, let

$$S(U) = \{Q : L_Q \in U\}.$$

- LEMMA 2: For any Borel set U, $\lambda^*(S(U)) = \nu(U)$.
- Given a ν-Martin-Löf test $\{V_n : n \in \mathbb{N}\}$ for L_Q, convert this into a Martin-Löf test $S(V_n) : n \in \mathbb{N}\}$ for Q.
- Then $\mu^*(S(V_n)) < 2^{-n}$ by Lemma 2,
- This is a c.e. test, since if $V_n = \bigcup_t I[\sigma_{n,t}]$, then $S(V_n) = \bigcup_t S(I[\sigma_{n,t}])$.
- If Q is Martin-Löf random, then for some n, $Q \notin S(V_n)$ and hence $L_Q \notin V_n$.
- Hence L_Q is ν-random.
- The proof for R_Q is similar

Proof of Lemma 2

- Induction on clopen sets
- For $U = \emptyset$, $S(U) = \emptyset$–both measure 0
- For $U = \{0,1\}^{\mathbb{N}}$, $S(U) = \{0,1\}^{\mathbb{N}} \times \mathcal{C}$– both measure 1
- Let $U_i = \{x : i^\frown x \in U\}$ and $Q_i = \{x : i^\frown x \in Q\}$.
- Case I: $(0) \in T_Q$: Then $Q \in S(U) \iff Q_0 \in U_0$.
- Case II: $(0) \notin T_Q$: Then $Q \in S(U) \iff Q_1 \in U_1$.
- Hence $\lambda^*(S(U)) = \frac{2}{3}\lambda^*(S(U_0)) + \frac{1}{3}\lambda^*(S(U_1))$
- By induction, this $= \frac{2}{3}\nu(U_0) + \frac{1}{3}\nu(U_1) = \nu(U)$.
- Induction on Borel Sets–it suffices that
- $S(\{0,1\}^{\mathbb{N}} - U) = \mathcal{C} - S(U)$
- $S(\bigcup_n U_n) = \bigcup_n S(U_n)$

A Converse to Theorem 1

- THEOREM 3:
  (i) If C is $\lambda_{\alpha\cdot\beta}$-random, then there exist A and B such that $C = A \cap B$, A is λ_α-random relative to B and B is λ_β-random relative to A.
  (ii) If C is $\lambda_{1-(1-\alpha)\cdot(1-\beta)}$-random, then there exist A and B such that $C = A \cup B$, A is λ_α-random relative to B and B is λ_β-random relative to A.
- PROOF of Part (i): Suppose that C is $\lambda_{\alpha\cdot\beta}$-random.
- Let $p = \frac{\alpha(1-\beta_i)}{1-\alpha\beta}$, let $q = \frac{\beta(1-\alpha)}{1-\alpha\beta}$, and let $r = \frac{1-\alpha)(1-\beta)}{1-\alpha\beta}$.
- Let $\gamma = (p, q, r)$ and let $g \in \{0, 1, 2\}^{\mathbb{N}}$ be γ-random.

Proof Continued

- Define A and B as follows.

$$\begin{aligned} i \in A &\iff i \in C \;\vee\; g(i) = 0; \\ i \in B &\iff i \in C \;\vee\; g(i) = 1; \end{aligned}$$

- Certainly $C = A \cup B$. It remains to show that A is α random and B is β random.
- Define the computable map $F : \{0,1\}^{\mathbb{N}} \otimes \{0,1,2\}^{\mathbb{N}} \to \{0,1\}^{\mathbb{N}}$ by

$$F(X, f) = X \cup \{i : f(i) = 0\}.$$

  so that $F(C, g) = A$.
- (C, g) is $(\alpha \cdot \beta) \oplus \gamma$ random by Van Lambalgen's Theorem.
- LEMMA 3: For any open set U, $\lambda_\gamma(F^{-1}(U)) = \lambda_\alpha(U)$.

Ghost Codes

- There is an equivalent formulation of randomness for closed sets via *ghost codes* (JLC 2007).
- Simply enumerate *all* strings from $\{0,1\}^*$ as $\tau_0, \tau_1, \dots$ and, if $\tau_n \in T_P$, we use $X(n) \in \{0,1,2\}$ to determine the branching below τ_n.
- When $\tau_n \notin T_P$, the ghost code $X(n)$ is not used in the definition of P.
- Thus the ghost code definition X of a random closed set contains a lot of unused information.
- The representation by ghost codes is not unique.
- *Any* closed set has a non-random representation –just let all of the unused ghost codes be 0.
- THEOREM (JLC 2007) A closed set is random iff there is a random representation by ghost codes.

Galton-Watson Trees

- Another formulation of via *Galton-Watson trees*. See Diamondstone–Kjos-Hanssen (CIE 2009)
- Let $\tau_n \in S_X$ iff $X(n) = 1$–independently for each n.
- Let $\tau \in T_X \iff (\forall \sigma \prec \tau)\sigma \in S_X$. The tree T_X is *Galton-Watson random* with survival parameter $\rho > \frac{1}{2}$ if X is λ_ρ random.
- $X(n)$ is not used if $\tau_n \notin T_X$.
- The representation by Galton-Watson trees is not unique and *any* closed set has a non-random representation.
- Q_X might be empty even for a random X. For $\rho < \frac{1}{2}$, $Q_X = \emptyset$ for all random X.
- THEOREM 4 (Axon,D-KH) For $p > \frac{1}{2}$, a (nonempty) closed set Q is Martin-Löf random with respect to the measure $\lambda_{1-p,1-p}$, iff there is a Galton-Watson tree T_X with survival parameter p such that $Q = Q_X$.

Unions of Random Sets

- THEOREM 5: If P_1 and P_2 are relatively random closed sets, then $P_1 \cup P_2$ is μ^*-random, where $\mu = \lambda_{\frac{1}{9},\frac{1}{9}}$.
- The intuition is that if $\sigma \in T_{P_1 \cup P_2}$, then σ will have a single extension (i) only when this is true in both P_1 and P_2, which is once out of 9 cases.
- PROOF OF THEOREM 5: Let ν be the product measure on $\{0, 1, 2\}^{\mathbb{N}} \times \{0, 1, 2\}^{\mathbb{N}}$.
- For any $U \subseteq \mathcal{C}$, let $S(U) = \{(X, Y) : Q_Y \cup Q_Y \in U\}$.
- LEMMA 3: For any Borel set $U \subseteq \mathcal{C}$, $\mu(S(U)) = \lambda^*(U)$.
- Now given a μ^*-Martin-Löf test $\{U_n : n \in \mathbb{N}\}$ for $P_1 \cup P_2$, let $P_i = Q_{X_i}$ for $i = 1, 2$.
- Then the sequence $\{S(U_n) : n \in \mathbb{N}\}$ is a ν-Martin-Löf test for $X_1 \oplus X_2$. Hence there exists n such that $X_1 \oplus X_2 \notin S(U_n)$, so that $P_1 \cap P_2 \notin U_n$.

Intersections of Random Sets

- THEOREM 6: If P and Q are relatively random closed sets, then $P \cap Q = \emptyset$.
- PROOF: We show that $\{(P, Q) : P \cap Q = \emptyset\}$ has measure one in the space of $\mathcal{C} \times \mathcal{C}$. Here we use the original representation from JLC07.
- Let $P_i = \{X : i^\frown X \in P\}$ and similarly for Q.
- $P \cap Q = \emptyset$ in the following cases.
- Case I: P only branches left and Q only branches right or vice versa. This has probability $\frac{2}{9}$.
- Case II: Not Case I but either P or Q has only a single branch (i) and $P_i \cap Q_i = \emptyset$. This has probability $\frac{2}{3} \cdot m$.
- Case IV: P and Q each have both branches and $P_i \cap Q_i = \emptyset$ for both $i = 0, 1$. This has probability $\frac{1}{9} \cdot m^2$.
- Hence $\frac{1}{9}m^2 + \frac{6}{9}m + \frac{2}{9} = m$.
  Thus $m^2 - 3m + 2 = (m - 1)(m - 2) = 0$ and it follows

Nonempty Intersections

- THEOREM 7: Let $0 < q < 1 - \frac{\sqrt{2}}{2}$ and suppose P_0 and P_1 are relatively $\lambda_{p,p}$ random closed sets, Then $P_0 \cap P_1$ is either empty or is $\lambda_{r,r}$ random with $r = 1 - q^2$.
- PROOF: Let P_0 and P_1 be relatively $\lambda_{q,q}$ random.
- Then by D-KH, P_0 and P_1 are the sets $([T_i])$ of paths through relatively random Galton-Watson trees T_0 and T_1 with survival parameter $p = 1 - q \geq \frac{\sqrt{2}}{2} > \frac{1}{2}$.
- That is, there are λ_p random sets X_0 and X_1 in $\{0,1\}^{\mathbb{N}}$ which define T_0 and T_1 as above.
- Now define $X = X_0 \cap X_1$. It follows from Theorem 1 that X is p^2 random. Note that $p^2 > \frac{1}{2}$.
- Let $\tau_n \in S_i \iff n \in X_i$.
  let $\tau_n \in S \iff n \in X$, so that $S = S_0 \cap S_1$
  and let $\tau \in T \iff (\forall \sigma \prec \tau)\sigma \in S$.
- Let $P = [T]$.

Existence of Nonempty Intersections

- THEOREM 8: If $q < 1 - \frac{\sqrt{2}}{2}$, then there exist relatively $\lambda_{q,q}$ random closed sets P and P such that $P \cap P \neq \emptyset$.
- PROOF: It suffice to show that the set of pairs with empty intersection has measure $m < 1$.
- Following the method of Theorem 6, we see that m is a root of the equation

$$(1 - 4q + 4q^2)m^2 + (4q - 6q^2 - 1)m + 2q^2) = 0$$

- This has roots $m = 1$ and $m = \frac{2q^2}{(1-2q)^2}$.
- It can be checked that if $q < 1 - \frac{\sqrt{2}}{2}$, then the second root is < 1 and will be the actual measure.

Members of Random CLosed Sets

- Some results from Barmpalias et al
- Every random closed set has a random element
- Random sets do not contain *n*-c.e. elements.
- There is a random closed set with no Δ^0_3 element.
- Every strong Δ^0_2 random closed set contains a Δ^0_3 element.
- For any random real A, there exists a random closed set Q with $A \in Q$.

Is X a Member of a Random Closed Set

- Let $p = 2^{-\gamma}$,
- Bjorn Kjos-Hanssen has shown the following: (and more!)
- THEOREM (KH): If X has effective Hausdorff dimension $> \gamma$, then X is a member of some $\lambda_{p,p}$ random closed set.
- THEOREM (Diamondstone-KH) If X is a member of some $\lambda_{p,p}$-random closed set, then X has effective Hausdorff dimension $\geq \gamma$.
- COROLLARY: Let $X \in \{0,1\}^{\mathbb{N}}$ be a λ_q-random. Then there exists a $\lambda_{p,p}$-random closed set Q containing X IFF $p \leq q \leq 1 - p$.
- We will examine the reverse question: Given a random closed set Q, for which q does Q contain a q-random element.
- The general problem is to determine the nature of the members of a random closed set.

Selectors

- A function $F : \mathcal{C} \to \{0,1\}^{\mathbb{N}}$ is a *selector* if $F(Q) \in Q$ for all nonempty closed sets Q.
- Given a measure μ^* on $\mathcal{C}$, a selector F will induce a measure μ_F on $\{0,1\}^{\mathbb{N}}$ where
- DEFINITION: $\mu_F(U) = \mu^*(F^{-1}(U))$ for all sets U.
- For example, if $F(Q) = L_Q$ and μ^* is the standard $(1/3, 1/3, 1/3)$ measure on $\mathcal{C}$, then μ_F is the measure $\lambda_{1/3}$ on $\{0,1\}^{\mathbb{N}}$.
- LEMMA 6: For any computable measure μ, if Q is a μ^*-random closed set, then $F(Q)$ is a μ_F-random member of Q.
- PROOF OF LEMMA 6: It follows as in Lemma 2 that any Martin-Löf -μ_F test for $F(Q)$ can be converted to a Martin-Löf -μ^* test for Q.
  This shows again that L_Q is $\lambda_{\frac{1}{3}}$ random.

Uniform Selectors

- A function $F : \{0,1\}^{\mathbb{N}} \times \mathcal{C} \to \{0,1\}^{\mathbb{N}}$ is a *uniform selector* if $F(Y,Q) \in Q$ for all $X \in \{0,1\}^{\mathbb{N}}$ and all nonempty closed sets Q.
- Given measures ν on $\{0,1\}^{\mathbb{N}}$ and μ^* on $\mathcal{C}$, let $\nu \oplus \mu^*$ be the product measure on $\{0,1\}^{\mathbb{N}} \times \mathcal{C}$. Then a uniform selector F will induce a measure μ_F on $\{0,1\}^{\mathbb{N}}$ where
- $\mu_F(X) = (\nu \oplus \mu^*)(F^{-1}(X))$.
- LEMMA 7: Let λ and μ be computable measures and let F be a uniform selector as above. Then for any μ-random Q and any Y which is λ-random with respect to $H(Y,Q)$ is a λ_F-random member of Q.
- PROOF OF LEMMA 7: It follows as usual that any Martin-Löf -μ_F test for $F(Y,Q)$ can be converted to a $\nu \oplus \mu^*$ test for Q.

Example

- EXAMPLE: Let $H(Y, Q) = X$ be defined recursively so that given $X\lceil n \in T_Q$,
  $X(n) = i$ whenever $X\lceil n$ has unique extension i in Q and
  $X(n) = Y(n)$ when $X\lceil n$ has both extensions.
- Note that if Q is not perfect and $H(Y, Q)$ is isolated in Q, then $H(Y, Q)$ is still defined and only makes use of finitely much of Y.
- LEMMA 8: μ_H is standard Lebesgue measure.

Proof of Lemma 8

- PROOF OF LEMMA 8: Suffices to show for subbasic sets.
- Let $U = I[(0)]$, which has measure $\frac{1}{2}$.
  For $H(Y, Q) \in U$, either
  - (0) Q begins with (0) and Y is arbitrary
    –this has measure $1 \cdot \frac{1}{3} = \frac{1}{3}$; or ')
  - (1) Q begins with both (0) and (1) and $Y(0) = 0$
    –this has measure $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

  Hence $\mu_H(U) = \frac{1}{2}$ also.
  For $U = I[(1)]$, the proof is similar.

The Induction

- For the induction, let $|\sigma| = n$ with $\sigma(0) = 0$ without loss of generality and consider $\sigma' = (\sigma(1), \ldots, \sigma(n-1))$, and $Y' = Y(1), Y(2), \ldots)$
- $\lambda(U) = \frac{1}{2}\lambda(U')$
- $\mu_F(U) = \frac{1}{3} \cdot \mu_F(U') + \frac{1}{2} \cdot \frac{1}{3} \cdot \mu_F(U') = \frac{1}{2}\mu_F(U') = \frac{1}{2}\lambda(U)$ where the first summand is for Q with single branch (0) and the second is for both branches.

The Random Selector

- THEOREM 10: For any computable measure μ, if Q is a random closed set and Y is a random real, then $H(Y, Q)$ is a random member of Q.
- PROOF OF THEOREM 10: This follows from Lemmas 7 and 8.

The Natural Measure μ_Q

- Now fix a perfect closed set Q and let $H_Q : \{0,1\}^{\mathbb{N}} \to Q$ by $H_Q(Y) = H(Y, Q)$.
- H_Q is a homeomorphism.
- Define the "natural" measure μ_Q on the space Q by $\mu_Q(U) = \lambda(H_Q^{-1}(U)) = \mu_{H_Q}(U)$.
- THEOREM 11: For any random closed set Q, $\mu_Q(\{X \in Q : X \text{ is random}\} = 1$.
  –that is, almost all members of Q are Martin-Löf random.
  PROOF: Since Q is random, it follows that almost all Y are random relative to Q and thus $H_Q(Y)$ is random for almost all Y.

λ_p-Random Members of Random Sets

- THEOREM 12: For any random closed set Q and any p with $\frac{1}{3} \leq p \leq \frac{2}{3}$, Q contains a λ_p-random real.
- PROOF OF THEOREM 12: Fix p and let $q = 3p - 1$.
- Let $Y \in \{0,1\}^{\mathbb{N}}$ be λ_q-random relative to Q.
- LEMMA 9: μ_H equals $\lambda_{\frac{1}{3}}$
- $H_Q(Y) = X \in Q$ is λ_p random.

Non-Symmetric Randomness

- THEOREM 9 Revisited: Let Q be a $\lambda_{a,b}$-random closed set and let $b \leq p \leq 1 - a$. Then Q contains a λ_p-random element.
- INTUITION: The leftmost path is λ_b-random and the rightmost path is λ_{1-a} random.
- Let $q = \frac{p-b}{1-a-b}$ and let Z be q-random.
- Now let $X = H(Z, Q) \in Q$.
- CLAIM: X is p-random.
- Observe that $Y \oplus Q$ is μ-random, where $\mu = \lambda_q \oplus \lambda_{a,b}$. Hence the following Lemma suffices
- LEMMA 7: $\lambda_p = \mu_F$.

Asymptotic Density

- Let $lim_n card(A \cap n)/n$ be the *asymptotic density* of A. If A is λ_p-random, then A has asymptotic density $1 - p$.
- Hence for any p with $\frac{1}{3} \leq p \leq \frac{2}{3}$ and any random closed set Q, Q contains an element with asymptotic density p.
- THEOREM 13: There exists $p < \frac{1}{3}$ such that any random closed set Q contains an element of asymptotic density p.
- PROOF OF THEOREM 13: For any tree $T \subseteq \{0, 1\}^{\leq 3}$, let $M(T)$ be the leftmost path of T among those with the most 0's.
  Define the selector $F(Q) = Y$ recursively as follows.
- Let $T_0 = T \cap \{0, 1, 2\}^{\leq 3}$ and let $(Y(0), Y(1), Y(2)) = M(T_0)$.
- Given $y\lceil 3n$, let $T_n = \{\sigma \in \{0, 1\}^{\leq 3} : Y\lceil 3n^\frown\sigma \in T\}$ and let $(Y(3n), Y(3n+1), Y(3n+2)) = M(T_n)$.

Proof Continued

- By Lemma 6, any random closed set Q contains the λ_F-random path $M(Q)$.
- LEMMA 7: If X is λ_M-random, then X has asymptotic density $\frac{1}{3} - \frac{4}{729}$.
- PROOF: Analysis of the possible trees of height 3 shows that $M(T)$ is the leftmost path $L(T)$ except when $L(T) = 011$ and $M(T) = 100$, which occurs with probability $\frac{4}{243}$.
- Thus the expected number of 1's in $M(T)$ is $1 - \frac{4}{243}$ (out of 3).
- By the Law of Large Numbers (e.g. Chernoff's Lemma) any λ_M-random sequence will have this expected asymptotic density $\frac{1}{3}(1 - \frac{4}{243})$ of 1's.

Final Thoughts

- Union and intersection for closed sets can be inverted.
- QUESTION: How effective is the decomposition of random sets?
- QUESTION: What are the possible asymptotic densities of members of random closed sets.
  CONJECTURE: Not all densities are possible.
- QUESTION: What can we say about the members of a random closed set Q which are computable from Q –or just $HY, Q)$ for some computable Y.
- PROBLEM: Is there a random closed set Q and a noncomputable A such that $A \leq_T X$ for all $X \in Q$.
  NOTE: Such A has to be K-trivial (observation of Slaman).

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- THANK YOU