Main results until end of 2010

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1 Introduction

This is a concise summary of my work until the end of 2010.

The bibliography is not complete, but is sufficient to find the results and ideas discussed here.

1.1 Overview

My work was and is centered on the development of new concepts for non-monotonic and related logics, on the development of new techniques to solve representation problems for such logics, and on their actual solution.

- Concepts

  Among others, the following concepts were developped, they, and others, will be detailed below.

  - Perhaps the, in hindsight, most important decision I made was not to show representation directly by an equivalence of logic and structure, but to split the problem in two parts, introducing an intermediate level (see [Sch92]), distinguishing *algebraic* from *structural* semantics. This made the main results flexible, and also led immediately to consider *definability preservation* as an important property of model choice functions. Both are discussed in Section 2.2.1 and Section 2.2.2. This, in turn, led to consider *small exception sets* and approximation for representation problems for operators lacking definability preservation.
  - Reflection on the intuition behind "normally" led to consider (systems) of *weak filters*, and then to consider manipulation of *abstract size* of model sets, see Section 2.3.1. This, in turn, led to consider *modularity* and *independence* and *semantic interpolation* for non-monotonic logics, see Section 2.3.2 and see Section 2.2.3.
  - More loosely connected, I also considered *distance semantics* for theory revision (with D. Lehmann and M. Magidor), see Section 2.4.1, general *neighbourhood semantics* for deontic and other logics, see Section 2.6, refinement of the notion of smoothness to *essential smoothness*, see Section 2.3.3, and *justification* for "bold" logics, see Section 2.2.4.

- Techniques

  Among the techniques invented for representation results are:

  (1) separation of representation problems into a syntactic and a semantic problem (see above)
    - this brings definability problems in evidence, and
    - translates syntactic problems to problems of representing algebraic properties of the semantics

  (2) approximations for not definability preserving stuctures

  (3) for preferential structures, introduction of
    - choice functions to represent arbitrary preferential structures,
    - trees of such functions for representation of transitive structures,
    - coherent sequences for representation of "smooth" structures

  (4) suitable diagonalisation methods to show impossibility of representation in various cases.

  As most of these techniques are general, and algebraic in nature, they can be re-used in different contexts, and also bring to light the abstract problems and their structures. The interested reader will find many examples of my techniques in the book [Sch04].

- Results

  Using these concepts and techniques in various combinations and variants, I was able to show a certain number of completeness results or, in the negative case, impossibility of representation. With two exceptions, I worked for the technical part on my own, even if the final publications were coauthored.

> My research on non-monotonic and related logics was published in 4 books, 35 journal articles, 14 conference publications, and 3 book chapters. In all joint publications, my share is at least proportional to the inverse of the number of authors, but often considerably more important.

Unless said otherwise, my work was done on propositional logic.

1.2 Work 2007-2010

I began to work with Dov Gabbay (King's College, London). This turned out to be extremely fruitful, as we are quite complementary in work and approach. We first finished some work I had already begun before (cumulativity without closure of the domain, additive laws about size, comments on work by Booth et al., defeasible inheritance systems), investigated higher preferential structures, and then turned to interpolation problems and modularity. The latter is probably the most important part of our joint work, as it opens up many new avenues of research (first order structures, etc.). Our joint work resulted in the following publications: [GS08a], [GS08b], [GS08c], [GS08d], [GS08e], [GS08h], [GS09a], [GS09c], [GS09f], [GS10].

2 The concepts in detail

2.1 Notation

$M(\phi)$ will be the set of (classical) models of the classical formula ϕ.

$\phi \mid\!\sim \psi$ will say that ϕ entails ψ in some (new) logic, whereas $\models$ will express classical validity (and consequence in the usual abuse of notation), $\vdash$ classical consequence.

μ will denote a choice function, $\mu(X)$ will be the "best" elements of X, likewise $\mu(\phi) \subseteq M(\phi)$ will be the best models of ϕ if μ is defined on the model set for a given language.

2.2 General concepts

We briefly describe here our main concepts and their importance.

2.2.1 Algebraic and structural semantics

Preferential structures are like Kripke structures, a set of models with a binary relation, say $\prec$. The relation is, however, used differently, and as follows: We are interested in the "best" or "minimally abnormal" elements, defined by $\mu(X) := \{x \in X : \neg\exists y \in X.y \prec x\}$. The logic is then defined by $T \mid\!\sim \phi$ iff $\mu(M(T)) \subseteq M(\phi)$. But the definition suggests already that we need not consider model sets, X may be any set. Moreover, we may look at the abstract properties of μ, like $\mu(X) \subseteq X$, or, less trivial, $X \subseteq Y \Rightarrow \mu(Y) \cap X \subseteq \mu(X)$. Whereas a preferential structure is a *structural* semantics, the abstract properties of μ are an *algebraic* semantics. Separating the two introduces an intermediate level between proof theory and structural semantics, splitting representation problems in two subproblems, and bringing to light sometimes subtle difficulties. In particular, the connection between abstract and structural semantics is logic independent, so can be used in many contexts, whereas the connection between proof theory and abstract semantics is often either trivial, or plagued with definability problems.

2.2.2 Definability preservation and exception sets

In the finite case, all model sets are definable. In the infinite case, this is not true. Given any preferential structure or abstract choice function μ, it is by no means evident that $\mu(\phi)$ will again be exactly the set of models of some formula ψ. If $\mu(\phi)$ is always $M(\psi)$ for some ψ, we say that μ is definability preserving. If this is not the case, usual representation results may fail. Worse still, one can sometimes show that no "usual" characterization is possible, but one has to work with "small" sets of exceptions, i.e., we cannot have a precise result, only an approximation.

2.2.3 Semantic interpolation

During our work on interpolation for non-monotonic logic, we invented a new form of interpolation, *semantic interpolation.* This is a concept suitable for all, also classical, logics, with suitable semantics. Essentially, given $X \subseteq Y$, where X, Y are subsets of a set of sequences (think of models as sequences of True/False), an interpolant Z of X and Y is a "simple" set such that $X \subseteq Z \subseteq Y$. Simple (think of the variables occurring in both sets) means here that the product is restricted at most where both X and Y are restricted. An example: Consider sequences of T/F, of length 3. Let $X := \{\langle T,T\rangle\} \times \{T,F\} = \{\langle T,T,T\rangle, \langle T,T,F\rangle\}$ $Y := \{T,F\} \times \{T\} \times \{T,F\} \cup \{\langle F,F,F\rangle\}$, then $Z := \{T,F\} \times \{T\} \times \{T,F\}$ is such an interpolant, it is restricted only on the second coordinate. The somewhat surprising result is that any monotone or antitone logic, even if it has more than 2 truth values, has semantic interpolants, but the language may not be powerful enough to define them.

Non-monotonic logics are different, as they often have a downward and then upward movement: $\phi \mid\!\sim \psi$ iff $\mu(\phi) \subseteq M(\psi)$, $\mu(\phi) \subseteq M(\phi)$ is the downward part, $\mu(\phi) \subseteq M(\psi)$ the upward part.

2.2.4 Justification

In ongoing work, I will look at what I see as a new fundamental concept of logic, justification. Here is why: Non-monotonic logics were created to reason about the normal case. Yet, often this intention is blurred, and one does "quick and dirty" reasoning as if every case were normal. But, such "bold" logics are wrong, and we know it. So we need a *justification* to do as if all cases were normal. This justification may be, e.g., that exceptions are rare, and error is not too costly.

2.3 Preferential structures

As said above in Section 2.2.1, preferential structures focus on $\prec$ $-$minimal elements. There is a conceptually interesting, but in the logical properties either trivial or "impossible" variant, which works with what is true "in the limit": E.g., in the case of infinite descending chains of models, there might be no minimal models, i.e., it might be the case that $X \neq \emptyset$, but $\mu(X) = \emptyset$, leading to $\phi \mid\sim FALSE$, if $X = M(\phi)$. In such cases, it is better to consider the "limit". We define what an initial segment is (basically, it includes all sufficiently small elements), and define $\phi \mid\sim \psi$ iff ψ is true in some initial segment.

2.3.1 Abstract size

In colloquial language, a non-monotonic statement may have the form: "Normally, birds fly" We may interpret this as "Normal birds fly", "Most birds fly", etc. More formally, we may say that the elements of a "big" subset of the set of birds fly. This leads to an abstract notion of size ($big, small, medium$) which may be weaker than the notion of a filter, as the union of a finite number of small sets may be a big set. We call such systems of subsets weak filters.

Usual laws about non-monotonic logics can often be expressed as laws about abstract size of model sets by the following definition: $M(\phi \wedge \psi)$ is a big subset of $M(\phi)$ iff $\phi \mid\sim \psi$. E.g., the AND rule - $\phi \mid\sim \psi$ and $\phi \mid\sim \psi'$ entail $\phi \mid\sim (\psi \wedge \psi')$ - corresponds to the filter property - the intersection of two big subsets is still big, or, dually, the union of two small sets is small, "$small + small = small$". Classical weakening - $\phi \mid\sim \psi$ and $\psi \models \psi'$ entail $\phi \mid\sim \psi'$ - corresponds to the property that supersets of big sets are big.

More interesting are properties which involve changes of the base set. If X is a small subset of Y, and $Y \subseteq Y'$, then it is natural to assume that X is also a small subset of Y'. This corresponds to the following rule: $\phi \mid\sim \psi$ and $\phi \models \phi'$ entail $\phi' \mid\sim (\psi \vee (\phi' \wedge \neg\phi))$. A more subtle argument involves making the base set smaller. Let $X, X' \subseteq Y$ be small subsets (and $X \subseteq Y - X'$), then we postulate that $X \subseteq Y - X'$ is also a small subset. The justification is that changing the base set by a small amount will not change sizes of subsets. This corresponds to (finite) Cumulativity: $\phi \mid\sim \psi$, $\phi \mid\sim \psi' \Rightarrow \phi \wedge \psi \mid\sim \psi'$.

A systematic investigation shows that one can derive many of the usual laws of non-monotonic logics by playing with plausible *additive* laws about size - and even detect new laws this way, filling in holes in a systematic picture. There is one exception which does not really fit in, Rational Monotony, $\phi \mid\sim \psi$, $\phi \not\mid\sim \neg\psi' \Rightarrow \phi \wedge \psi' \mid\sim \psi$, which is best seen as a *multiplicative* law, roughly, $medium * medium = medium$, where "medium" is neither "big" nor "small".

But there is also a connection between multiplicative laws about size and interpolation for non-monotonic logic. We were able to show that the law $big * big = big$, more precisely, if X and Y are sets of sequences, and $X \times Y$ their product, then

$(\mu*)$ $\mu(X \times Y) = \mu(X) \times \mu(Y)$

holds, entails interpolation of the form $\phi \mid\sim \alpha \mid\sim \psi$, where α is a suitable "simple" formula, i.e. containing only language elements common to ϕ and ψ. (We have also investigated other forms of non-monotonic interpolation, and suitable "Hamming" relations, generating such multiplicative size laws.)

In summary, considering non-monotonic logics as rules on manipulation of abstract size reduces them to very few basic properties, whose combination and modification by adjusting simple parameters can fully describe many of them, and even generate new logics. It thus seems to be the right level of abstraction for the consideration of these logics.

2.3.2 Modularity

Interpolation is a form of modularity. Essentially, it means that we can calculate consequences in sublanguages, and then put the results together. This is trivial in classical language (thus the semantical interpolation result for monotone and antitone logics), but not at all clear for non-monotonic logics. Consider a propositional language of two variables, p and q. We may have the following preference relation on its models: $pq \prec \neg p \neg q \prec \neg pq \prec p \neg q$, which cannot be recovered from a model order on the sublanguage $\{p\}$ and $\{q\}$. (A "nice" order would originate from $p \prec \neg p$ and $q \prec \neg q$, giving pq as the overall best model, and $\neg p \neg q$ as the overall worst model.)

But modularity is an extremely important property, at least as important as other regularities like AND, Cumulativity, etc., yet it has been largely ignored so far. Multiplication of size gives us a handle on it, and we intend to continue its investigation. The idea of multiplicative laws has also paved the way to a new look on first order preferential structures and "subideal cases", this is work in progress.

(The first incitation to consider independence anew came through work by R. Parikh and co-authors on modular theory revision.)

2.3.3 Essential smoothness

During work on higher preferential structures - a special case of Gabbay's reactive diagrams (in ususal preferential diagrams, elements may be "attacked" by better elements, in higher preferential diagrams, attacks may also be attacked, etc.) - it became clear that the usual concept of smoothness is too strong, the more suitable concept is "essential" smoothness, which can be described as follows: Usual smoothness postulates that, if an element in a structure is attacked, then it has to be attacked from a point which itself is not attacked. Essential smoothness restricts this to elements of which no "copy" survives - for "copies" and other details see, e.g., [Sch04].

2.4 Theory revision

AGM (for Alchourron, Gardenfors, Makinson) revision is an approach to adding new information while preserving consistency. More precisely, given a consistent theory T, and new information ϕ, which is also consistent, but not together with T, we want to modify T "minimally" to T' such that $T' \cup \{\phi\}$ is now consistent.

2.4.1 Abstract distance

In joint work with D. Lehmann and M. Magidor, we introduced abstract distance semantics for (AGM) theory revision, and have shown soundness and completeness. This distance semantics is similar to the Stalnaker/Lewis semantics for counterfactual conditionals, except that we consider the *globally* closest models, and not those which are *individually* closest to the first model set. Closeness codes minimal change in our approach. We were able to show that all AGM axioms hold in our semantics. But our system is considerably stronger, as we can also change the original T, which stayes fixed in the AGM system. This has far reaching consequences, as I was able to show that our semantics does not have finite axiomatizations any more - whereas AGM revision has.

In general, it seems much easier to work on the distance semantics side than on the proof theoretical side; often what is a complicated argument on the latter, is a triviality on the former.

2.5 Inheritance theory

Inheritance diagrams are deceptively simple. The basic problems can be illustrated by two little diagrams. Here, simple arrows denote strict or "soft" inclusion (or membership), crossed arrows denote strict or "soft" inclusion in the *complement*, thus it is a strong negation. ("Soft" means: perhaps with some exceptions.)

Tweety diagram

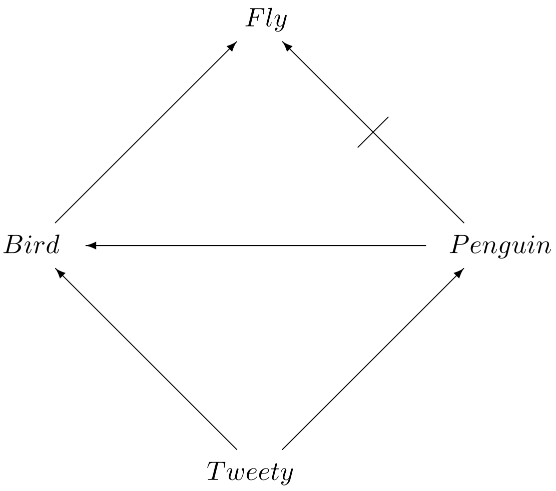

Diagram 2.1

The Nixon Diamond

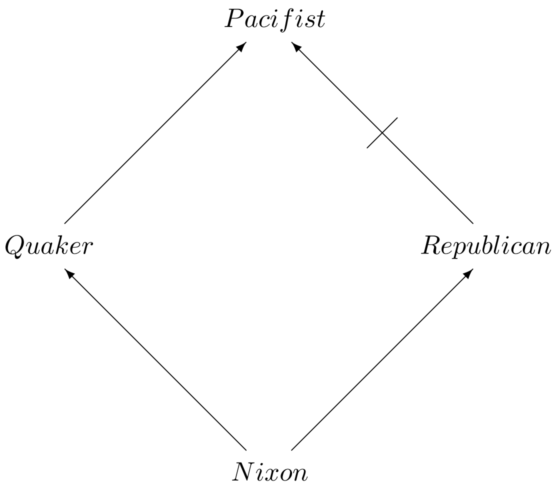

Diagram 2.2

In the Tweety diagram, the information attached to the fact that Tweety is a penguin is considered stronger than the information attached to the fact that Tweety is a bird. The criterion here is specificity, thus we conclude that Tweety will not fly.

In the Nixon diamond, there is no such criterion, and we cannot decide between the contradictory information: As a Republican, Nixon should be a hawk, as a Quaker, he should be a pacifist. We either conclude nothing (direct scepticism), or branch into two possibilities (extensions), and consider the intersections of extensions.

I have shown that the two approaches (direct scepticism vs. intersection of extensions) are fundamentally different, even if we are allowed to work with arbitrarily many finite truth values.

In recent work, I have taken a more radical approach, and seen such systems as constructing the language successively - i.e., we do *not* work in a fixed language from the outset, but construct the language by what is reachable through valid paths -, and, in addition, as constructing a truth value structure, where specificity decides on the relative strength. I think that this approach - though at first sight perhaps too radical - is the right one, and does justice to these networks.

2.6 Neighbourhood semantics for deontic and other logics

I have considered neighbourhood semantics for various logics, like deontic logic (not necessarily what is “best”, but also what is “sufficiently” good), but also as an abstract semantics for the “limit version” of preferential structures.

As the problem presents itself in several contexts, I looked at it abstractly, defining and investigating several reasonable definitions and properties. In the deontic case, this approach solves the Ross paradox (which shows that deontic postulates may not be weakened arbitrarily, e.g., “you should not steal” should not be weakened to: “you should (not steal) or (kill)” - attention to the distribution of the “not”).

3 Results

For older results, I do not always give here the first publication, often only a book summarizing the results. Results which were achieved in substantial cooperation with other authors are maked as such.

3.1 General results

The results on interpolation were shown in [GS09c] and [GS10]. Given two sets of sequences, $X \subseteq Y$, it is shown that there is *always* a suitable (semantic) interpolant A with $X \subseteq A \subseteq Y$, moreover, there are minimal and maximal A and A' such that $X \subseteq A \subseteq A' \subseteq Y$, both "simple", and (meta-) definable in a natural way. In addition, both are universal in that they depend only on one of X or Y, and the language of the other. We can make them definable in the object language through the introduction of new operators. Thus, semantic interpolation becomes also syntactic interpolation. This result generalizes immediately to more than two truth values. We also consider here the special situation of finite Goedel logics, and various new operators to assure sufficient expressive power of the language to make interpolants definable.

3.2 Abstract size, and defaults as generalized quantifiers

We first used abstract size to give a semantics to defeasible inheritance formalisms, see [Sch97-2] and and [Sch90]. This was further developped into a weak filter semantics for a new, generalized quantifier, "almost all", for first order logic, soundness and completeness were shown, see [Sch97-2] and [Sch95-1]. We then showed equivalence of three abstract size systems by Ben-David/Ben-Eliyahu, Friedman/Halpern, and myself, see [Sch04].

A systematic investigation of *additive* size rules was done in [GS09f] and [GS08c], where we showed how to construct various logical rules of non-monotonic logics from simple reasoning about abstract size. This translation is very straightforward and systematic, resulting in tables of correspondence between the two sides, and giving a very transparent picture of various logical systems. Considering *multiplication* of abstract size allowed us, on the one hand, to fit the non-monotonic rule of rational monotony, $\phi \mid\!\sim \psi$, $\phi \not\mid\!\sim \neg\psi' \Rightarrow \phi \wedge \psi' \mid\!\sim \psi$, better into the overall picture, and, more importantly, to an algebraic definition of independence or modularity for non-monotonic logics and conditions for (semantic) interpolation, $(\mu *)$ above, see [GS10].

3.3 Preferential structures

Many of my results on preferential structures fit into the following schema:

On the one hand, we have preferential structures with perhaps certain additional properties, like transitivity, smoothness, etc., the structural semantics. In the middle, we have properties of the model choice function μ, the main such property is

(μPR) $X \subseteq Y \Rightarrow \mu(Y) \cap X \subseteq \mu(X)$,

this is the algebraic semantics. On the other side, we have logical properties, like infinite conditionalization:

(PR) $\overline{\overline{T \cup T'}} \subseteq \overline{\overline{\overline{T}} \cup T'}$

(here, $\overline{\overline{T}} := \{\phi : T \mid\!\sim \phi\}$, and $\overline{T} := \{\phi : T \vdash \phi\}$ in the author's notation).

Representation results describe then correspondences between logic, algebraic and structural semantics.

I have shown that:

General and transitive preferential structures are fully described by above algebraic condition (μPR) and the trivial condition $\mu(X) \subseteq X$. This was done in [Sch92] with a somewhat ad hoc proof for the transitive case, which was later improved be indexing by trees, see [Sch04].

Smooth and smooth and transitive structures are fully characterized by above conditions and semantic cumulativity:

(μCUM) $\mu(X) \subseteq Y \subseteq X \Rightarrow \mu(X) = \mu(Y)$

provided the domain is closed under finite unions. This is shown in [Sch04]. Work on Plausibility logic, where the domain is *not* closed under finite unions, and which showed that they may not be representable by smooth structures gave the first indication that the closure condition is necessary, see [Sch04]. Worse still, it can be shown that there are infinitely many different variants of cumulativity, if the domain is not closed, see [GS08a] and [GS09f]. Further results were shown on various forms of ranked structures, and their corresponding algebraic description, see [Sch04] and [GS09f].

For connecting the algebraic to the logical properties, $definablity\ preservation$ (dp) of the μ-function is extremely important. First, it was shown that the basic law (PR) does not necessarily hold in structures which do not preserve definability, see [Sch92] and [Sch04]. Second, if (dp) is satisfied, then there is an easy correspondence between logical and algebraic properties, e.g., (PR) corresponds to (μPR), see [Sch92] and [Sch04]. Analogous correspondences hold for (μCUM) and

(CUM) $T \subseteq \overline{T'} \subseteq \overline{\overline{T}} \Rightarrow \overline{\overline{T}} = \overline{\overline{T'}}$, the infinite version of Cumulativity,

etc., see [Sch04]. Third, in the absence of (dp), it can be shown that there is no logical characterization of the ususal form of *any* fixed (even infinite) size possible, but there are characterizations using "small" exception sets and approximation, see [Sch04]. Thus, definability preservation is really a central condition in representation problems for such structures.

For the limit version of preferential structures, it was shown that in many cases, they do not differ from the minimal version. More precisely, when, e.g., we examine only the consequences of formulas, and not necessarily full theories, then the logical behaviour of such structures corresponds to the behaviour of another structure in the minimal reading. On the other hand, the fully general situation is the same as the situation without definability preservation: no usual characterization is possible. See [Sch04].

Higher and essentially smooth higher preferential structures were characterized in [GS09f] and [GS08b]. The main interest is that we can have many usual rules of non-monotonic logics, including cumulativity, but without (PR) in a natural and very interesting semantics, which is close to general argumentation: we have attacks and attacks against attacks, etc.

We have investigated in [GS10], partly with D. Gabbay, semantic interpolation for non-monotonic logics based on model choice functions, see Section 2.3.1 above. Interpolation of $\phi \mid\!\sim \psi$ can be interpreted in three different ways: there is "simple" α such that $\phi \mid\!\sim \alpha \mid\!\sim \psi$, or $\phi \vdash \alpha \mid\!\sim \psi$, or $\phi \mid\!\sim \alpha \vdash \psi$. The last case has a full characterization, basically, that $\mu(\phi)$ is not more complicated than $M(\phi)$ is. The second case has sufficient conditions, but which seem often too strong. The most interesting case seems to be the first one, we have again only a sufficient, but very nice condition, already mentioned above, $(\mu *)$, see Section 2.3.1.

Finally, various other results include a proof that the extension of a result by S. Kraus, D. Lehmann, M. Magidor to the infinite case is impossible, see [Sch04] and [Sch92], a construction of a non-smooth model of cumulativity, see [Sch04], and an investigation of an alternative construction of preferential structures, see [Sch04].

3.4 Theory revision

I first gave a probability based semantics for AGM theory revision, see [Sch04]. In cooperation with D. Lehmann and M. Magidor, we gave a distance based semantics for (an extension of) AGM revision, and showed soundness and completeness. This characterization involves an arbitrarily big loop condition, and I have later shown that our extension cannot be characterized by any usual finite axiomatization - in contrast to the AGM situation, which is described by 8 simple axioms. The basic reason is that revision is not powerful enough to observe and compare all possible distances. Again, our results are only valid under a suitable assumption about definability preservation, and I was also able to show that, without this assumption, there is *no* usual characterization of any size possible, but we can work with "small" exception sets. (The general limit variant has no usual characterization either, but many interesting cases have the same behaviour as the minimal variant.) All these results are presented in [Sch04]. Revision by weaker notions of distance are briefly investigated in [GS10].

3.5 Theory update and counterfactual conditionals

We have shown that one can work in the Stalnaker/Lewis semantics for counterfactual conditionals without loss of generality with *one global* distance. Conceptually, it is permitted to work with individually distances, centered around each world, but using repetition of worlds, we can construct a semantics with one global distance, which has logically exactly the same properties, see [Sch04].

In [BLS99], we have investigated a semantics for iterated update and shown soundness and completenss (with S. Berger and D. Lehmann). The conditions are complicated by the fact that - analogous to the not definability preserving situations - the chosen sets might not be of the "nice" form the original sets have. This is a recurrent problem, which is also seen in the discussion of more general revisions in [GS10].

We have investigated update by minimal sums (of distances), and given a characterization using an old algorithm due to Farkas. This gives an arbitrarily big (finite) characterization, and we were able to show that there is no finite characterization possible for this form of update. (The situation for a characterization of "between" and "behind" is similar.) All these results are discussed in [Sch04].

3.6 Inheritance theory

Our main formal result for inheritance theory is the proof that the intersections of extensions and the directly sceptical approach are fundamentally different, and even the introduction of arbitrarily, but finitely, many values for the validity of paths cannot make them equivalent, see [Sch97-2].

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